

ON A UNIVERSAL DIFFERENTIAL EQUATION FOR THE ANALYTIC TERMS OF C^∞ -SUPERPOSITIONS ON THE REAL LINE

Carsten Elsner

Journal of Mathematical Analysis and Applications 295 (2004), 315-330

Let $\omega \in C(\mathbb{R}) \rightarrow \mathbb{R}_{>0}$ denote a bounded continuous weight function taking positive values everywhere, such that

$$\int_{-\infty}^{\infty} \omega(x) dx = 1$$

holds. Then, by

$$\|g\|_\omega := \int_{-\infty}^{\infty} \omega(x)|g(x)| dx \quad (g \in C(\mathbb{R})),$$

a norm for continuous functions g on the real line is given.

Theorem 1 *There exists a nontrivial autonomous algebraic differential equation $P = 0$ of order at most 7, where P denotes an effectively computable polynomial in at most eight variables, having the following property.*

Let $f \in C(\mathbb{R}) \rightarrow \mathbb{R}$ be some continuous function defined on the real line and let ε be some arbitrary positive number. Then a superposition $H \in C^\infty(\mathbb{R})$ of analytic functions $H_r \in C^\omega(\mathbb{R})$ exists, say

$$H(x) = \sum_{-\infty < r < \infty} H_r(x) \quad (x \in \mathbb{R}),$$

such that $\|f - H\|_\omega < \varepsilon$ holds, and every analytic function H_r solves the above universal differential equation. Moreover, every analytic function H_r on \mathbb{R} is an entire function on \mathbb{C} .