

# ALGEBRAIC RELATIONS FOR RECIPROCAL SUMS OF ODD TERMS IN FIBONACCI NUMBERS

Carsten Elsner, Shun Shimomura, and Iekata Shiokawa

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In this paper, we prove the algebraic independence of the reciprocal sums of odd terms in Fibonacci numbers

$$\sum_{n=1}^{\infty} \frac{1}{F_{2n-1}}, \quad \sum_{n=1}^{\infty} \frac{1}{F_{2n-1}^2}, \quad \sum_{n=1}^{\infty} \frac{1}{F_{2n-1}^3},$$

and write each

$$\sum_{n=1}^{\infty} \frac{1}{F_{2n-1}^s} \quad (s \geq 4)$$

as an explicit rational function of these three numbers over  $\mathbb{Q}$ . Similar results are obtained for various series including the reciprocal sums of odd terms in Lucas numbers.

**Key words:** Algebraic independence, Fibonacci numbers, Lucas numbers, Jacobian elliptic functions, Ramanujan functions,  $q$ -series, Nesterenko's theorem.