ALGEBRAIC INDEPENDENCE RESULTS FOR THE SIXTEEN FAMILIES OF q - SERIES

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Ramanujan Journal 22, no. 3 (2010), 315–344

The sixteen families of q-series containing the Ramanujan functions were listed by I. J. Zucker (1979), which are generated from the Fourier series expansions of the Jacobian elliptic functions or some of their squares. This paper discusses algebraic independence properties for these q-series. We determine all the sets of q-series such that, at each algebraic point, the values of the q-series in the set are algebraically independent over $\mathbb{Q}$. We also present several algebraic relations over $\mathbb{Q}$ for two or three of these q-series.

MR 2000 Subject Classification: 11J81

Key words: Algebraic independence, Jacobian elliptic functions, Ramanujan functions, q-series, Nesterenko's theorem.