

SERIES OF ERROR TERMS FOR RATIONAL APPROXIMATIONS OF IRRATIONAL NUMBERS

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Let p_n/q_n be the n -th convergent of a real irrational number α , and let $\varepsilon_n = \alpha q_n - p_n$. In this paper we investigate various sums of the type $\sum_m \varepsilon_m$, $\sum_m |\varepsilon_m|$, $\sum_m \varepsilon_m x^m$. The main subject of the paper are bounds for these sums. Particularly, we investigate the behaviour of such sums when α is a quadratic surd. Most significant properties of the error sums depend essentially on Fibonacci numbers or on related numbers.

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