

ALGEBRAIC INDEPENDENCE OF CERTAIN NUMBERS RELATED TO MODULAR FUNCTIONS

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(submitted paper)

In previous papers the authors established a method how to decide on the algebraic independence of a set $\{y_1, \dots, y_n\}$ when these numbers are connected with a set $\{x_1, \dots, x_n\}$ of algebraic independent parameters by a system $f_i(x_1, \dots, x_n, y_1, \dots, y_n) = 0$ ($i = 1, 2, \dots, n$) of rational functions. Constructing algebraic independent parameters by Nesterenko's theorem, the authors successfully applied their method to reciprocal sums of Fibonacci numbers and determined all the algebraic relations between three q -series belonging to one of the sixteen families of q -series introduced by Ramanujan.

In this paper we first give a short proof of Nesterenko's theorem on the algebraic independence of π , $e^{\pi\sqrt{d}}$ and a product of Gamma-values $\Gamma(m/n)$ at rational points m/n . Then we apply the method mentioned above to various sets of numbers. Our algebraic independence results include among others the coefficients of the series expansion of the Heumann-Lambda function, the values $P(q^r)$, $Q(q^r)$, and $R(q^r)$ of the Ramanujan functions P , Q , and R , for $q \in \overline{\mathbb{Q}}$ with $0 < |q| < 1$ and $r = 1, 2, 3, 5, 7, 10$, and the values given by reciprocal sums of polynomials.

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