SHIFTING ALGEBRAIC INDEPENDENCE MEASURES

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Let $x_1, \ldots, x_n$ be algebraically independent over $\mathbb{Q}$, and let $y_1, \ldots, y_n$ satisfy the system $P_\nu(x_1, \ldots, x_n, y_1, \ldots, y_n) = 0$ with $P_\nu \in \mathbb{Q}(X_1, \ldots, X_n, Y_1, \ldots, Y_n)$ for $(1 \leq \nu \leq n)$. In joint work with Sh. Shimomura and I. Shiokawa the first author has found sufficient conditions to decide on the algebraic independence of $y_1, \ldots, y_n$ over $\mathbb{Q}$ by a determinant criterion. In this paper we find an algebraic independence measure for the numbers $y_1, \ldots, y_n$ by using an algebraic independence measure for $x_1, \ldots, x_n$. Our method is essentially based on estimates for heights of polynomials. In particular, we prove a quantitative supplement to the fundamental theorem on symmetric functions. We give two applications of our results.

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