We first consider continuous functions \( f \) on \( \mathbb{R} \) where both the limits
\[
\lim_{x \to -\infty} f(x) \quad \text{and} \quad \lim_{x \to +\infty} f(x)
\]
exist in \( \mathbb{R} \). One gets:

**Theorem 1** There exists a nontrivial sixth-order ADE of the form
\[
P(x; y', y'', \ldots, y^{(6)}) = 0
\]
such that any real continuous function \( f \) on the real line can be uniformly approximated by the real analytic solutions of (2), provided that the limits in (1) exist in \( \mathbb{R} \).

**Corollary 1** There exists a nontrivial ADE of the form
\[
P(y', y'', \ldots, y^{(7)}) = 0
\]
such that the real-analytic solutions have the same property as the real-analytic solutions of the ADE (2).

**Theorem 2** There exists a nontrivial AFE of the form
\[
P(y'(x), y'(x + \log 2), \ldots, y'(x + 5\log 2)) = 0
\]
such that for any compact interval \( I \) the real-analytic solutions of (3) (defined on the whole real line) are dense in \( C(I) \).

**Theorem 3** Let \( n \) denote some positive integer, and let \( P \) be some non-zero polynomial having real coefficients. Then, for every positive real number \( \Delta \) and any compact interval \( I \subset \mathbb{R} \) satisfying \( |I| > 2n\Delta \), the continuous solutions \( g \) of the AFE
\[
P(g(x), g(x + \Delta), \ldots, g(x + n\Delta)) = 0 \quad (x \pm n\Delta \in I)
\]
are not dense in \( C(I) \).
By $M(\mathbb{C})$ we denote the set of meromorphic functions $f$ defined on $\mathbb{C}$ such that additionally two conditions are satisfied:

(i) $f(z)$ is analytically at every point $z$ with $\Im z = 0, \pm 2\pi, \pm 4\pi$.

(ii) $f(z)$ takes real values at points $z$ from the real axis.

Particularly every function $f(z)$ from $M(\mathbb{C})$, where $z$ is restricted on the real line, represents a real-valued analytical function from $C^\omega(\mathbb{R})$, and there is some real-valued analytical antiderivative defined on the real axis.

**Theorem 4** Let $I$ denote a compact interval. Then every function from $C(I)$ can be uniformly approximated by real-valued analytic functions $y$ defined on $I$, such that $y' \in M(\mathbb{C})$ holds and $y'$ satisfies the AFE

$$y_1y_2^2y_3y_5 - y_1y_2^2y_4 - y_1y_2y_4y_5 + y_1y_2y_3y_4 + y_1y_2y_1^2y_5$$

$$-y_1y_3y_4^2y_5 - y_2^2y_3y_4y_5 + y_2^2y_1^2y_5 = 0 \quad (4)$$

with

$$y_1 := y'(x - 4\pi i), \quad y_2 := y'(x - 2\pi i), \quad y_3 := y'(x),$$

$$y_4 := y'(x + 2\pi i), \quad y_5 := y'(x + 4\pi i).$$

The identity in (4) represents an AFE of order one for real-valued analytical functions on arbitrary compact intervals $I$. 