ÜBER GEWISSE LÖSUNGEN UNIVERSELLER DIFFERENTIALGLEICHUNGEN IN ALGEBRAISCHEN PUNKTEN

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It is shown that an effectively computable algebraic differential equation of order five exists such that on the one hand every continuous function \( f : \mathbb{R} \to \mathbb{R} \) can be approximated uniformly on \( \mathbb{R} \) by a sequence \( y_n(x) \) of \( C^\infty(\mathbb{R}) \)-solutions. On the other hand let \( 1 \leq k_1 < k_2 < \ldots < k_r = K \) be integers. Then, for every index \( n \) and for numbers \( y_n^{(k_1)}(\tau), \ldots, y_n^{(k_r)}(\tau) \), a linear transcendence measure exists which is effectively computable, i.e. a lower bound exists for

\[
\sum_{\rho=1}^{r} \psi_\rho y_n^{(k_\rho)}(\tau)
\]

with real algebraic numbers \( \psi_1, \ldots, \psi_r \). This measure depends on degrees and heights of \( \tau \) and \( \psi_1, \ldots, \psi_r \), but also on the modulus of continuity of the function \( f \) within an interval containing \( \tau \) and on the distance between \( y_n(x) \) and \( f(x) \) on \( \mathbb{R} \). The proof requires extensive estimates of degrees and heights of polynomials and algebraic numbers which are connected with the explicit solutions \( y_n(x) \) of the universal differential equation. The linear transcendence measure is finally given by a quantitative version of Baker’s theorem on linear forms in logarithms.