ON PRIME-DETECTING SEQUENCES FROM APÉRY’S RECURRENCE FORMULAE FOR $\zeta(3)$ and $\zeta(2)$

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We consider the linear three-term recurrence formula

$$X_n = (34(n - 1)^3 + 51(n - 1)^2 + 27(n - 1) + 5)X_{n-1} - (n - 1)^6X_{n-2} \quad (n \geq 2)$$

corresponding to Apéry’s non-regular continued fraction for $\zeta(3)$. It is shown that integer sequences $(X_n)_{n \geq 0}$ with $5X_0 \neq X_1$ satisfying the above relation are prime-detecting, i.e. $X_n \not\equiv 0 \pmod{n}$ if and only if $n$ is a prime not dividing $|5X_0 - X_1|$. Similar results are given for integer sequences satisfying the recurrence formula

$$X_n = (11(x - 1)^2 + 11(x - 1) + 3)X_{n-1} + (n - 1)^4X_{n-2} \quad (n \geq 2)$$

corresponding to Apéry’s non-regular continued fraction for $\zeta(2)$ and for sequences related to log $2$.


Key words: recurrences, sequences mod $m$, primes, continued fractions