ON CONVERGENTS FORMED FROM
DIOPHANTINE EQUATIONS

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We compute upper and lower bounds for the approximation of certain values \( \xi \) of hyperbolic functions by rationals \( x/y \) such that \( x, y \) satisfy Diophantine equations. We show that there are infinitely many coprime integers \( x, y \) such that

\[
|y\xi - x| \ll \frac{\log \log y}{\log y}
\]

and a Diophantine equation holds simultaneously for some integer \( z \). Conversely, all positive integers \( x, y \) with \( y \geq c_0 \) solving the Diophantine equation satisfy

\[
|y\xi - x| \gg \frac{\log \log y}{\log y}.
\]

Moreover, we approximate \( \sin(\pi \alpha) \) and \( \cos(\pi \alpha) \) by rationals in connection with solutions of a quadratic diophantine equation when \( \tan(\pi \alpha/2) \) is a Liouville number.

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